COMPUTATIONAL SOLUTION OF TEMPERATURE DISTRIBUTION IN A THIN ROD OVER A GIVEN INTERVAL

\[ I = \{ x \mid 0 < x < 1 \} \]

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ABSTRACT
In this paper, two analytical–numerical algorithms are formulated based on homotopy perturbation method and new iterative method to obtain numerical solution for temperature distribution \( u(x, t) \) in a thin rod over a given finite interval. The effects of different parameters such as the coefficient \( \beta \) which accounts for the heat loss and the diffusivity constant \( \phi \) are examined when initial temperature distribution \( u(x, 0) \) (trigonometry and algebraic functions) are considered. The error in both algorithms approaches to zero as the computational length \( P \) increases. The proposed algorithms have been demonstrated to be quite flexible, robust and accurate. Thus, the algorithms are established as good numerical tools to solve several problems in applied mathematics and other related field of sciences.

Keywords: Homotopy Perturbation Method, New Iterative Method, Temperature Distribution, Diffusivity Constant, Computational Algorithms

INTRODUCTION
The heat equations model the flow of heat through materials. They arise from interpretation of models of physical phenomena to reality in applied sciences and engineering such as in fluid mechanics, thermodynamics, vibration analysis, electrical engineering and applied physics. In a thin rod with non-uniform temperature, thermal energy is transferred from regions of higher temperature to regions of lower temperature. Heat diffusion equation is a parabolic partial differential equation (PDE) which describes the heat distribution in a given region and provides the basic tool for heat conduction analysis. The numerical solutions for this type of equation provide understanding knowledge of the temperature distribution which may then be used with Fourier’s law to determine the heat flux as elucidated in the works of Guenther and Lee (1996), Cannon and Browder (1984) and Bergman et al. (2011). Seeking of analytic-numerical methods has gained the interest of researchers for finding approximate solutions to PDEs. This interest is driven by demand of applications both in industries and sciences which leads to investigate analytical and numerical methods for solving initial and boundary value problems as expressed by Cheniguel (2014). In this paper, the temperature distribution \( u(x, t) \) is sought in a thin rod over the finite interval \( I = \{ x \mid 0 < x < 1 \} \) whose lateral surface is not insulated. The rod is experiencing a heat loss proportional to the difference between the rod temperature and the surrounding temperature at zero degrees. Adopting the form of the homogeneous partial differential equation expressed by Articolo (2009), we have the second order heat equation as:

![Figure 1: A thin rod of length L not insulated on its sides along x-coordinate at time t.](image-url)
\[
\frac{\partial u(x,t)}{\partial t} = \phi \left( \frac{\partial^2 u(x,t)}{\partial x^2} \right) - \beta u(x,t) \quad k, \beta > 0, \quad 0 \leq x \leq 1, \quad 0 \leq t \geq T
\]  
(1)

with the initial temperature used in the works of Articolo (2009) and Moore (2005):

\[
u(x,0) = \left\{ \begin{array}{l}
4 \sin(x) + 2 \sin(2x) + 7 \sin(3x) \\
1 - \frac{x^2}{3}
\end{array} \right.
\]  
(2)

The initial and boundary conditions are:

\[
\begin{cases}
u_t(x,0) = 0 \\
u(1,t) + \nu(1,t) = 0
\end{cases}
\]  
(3)

The left end of the rod is insulated and the right end is losing heat by convection into a zero temperature surrounding where coefficient \( \beta \) accounts for the heat loss and \( \phi \) is called the thermal diffusivity of the medium under consideration. This constant is equal:

\[
\phi = \frac{k}{\rho c_p}
\]  
(4)

where \( c_p \) the specific heat of the medium at constant pressure, \( \rho \) is the mass density and \( K \) is the thermal conductivity of the medium. By convention, it could be said that heat is a diffusion process whereby heat flows from high-temperature regions to low-temperature regions similar to how salt in a water solution diffuses from high-concentration regions to low-concentration regions. The magnitude of the thermal diffusivity \( \phi \) is an indication of the ability of the medium to conduct heat from one region to another. In this paper, \( u(x,t) \) denotes the temperature at the point \( x \)-metres along the rod at time \( t \) (in seconds) and the authors are interested in how the temperature along the rod varies with time which is subjected to initial temperature distribution \( u(x,0) \) (in the forms of trigonometry, exponential and logarithm functions).

Recently, Wegrzyn-Skrzypczak and Skrzypczak (2017) presented the results of analytical and numerical solutions for the problem of heat transport in the rod of finite length employing Fourier series and Finite Element Method (FEM). They addressed the compatibility of both solutions and distribution of the temperature for selected time moments. Among many researchers who investigated the heat transfers in solid media is Makhtoumi (2017). He presented a numerical method for the heat equation over one dimensional rod region. Al-Huniti et al. (2002) studied dynamic response of a rod due to a moving heat source under the hyperbolic heat conduction model. Also, Haggkvist (2009) considered a practical and theoretical approach to the problem of plate thermometer, which is a robust and simple instrument to be used to handle fierce environmental conditions, as a mean of calculating incident heat radiation. He (1999) presented a study for the numerical solution of three-dimensional second order partial differential equation that occurs in a thin plate vibration behaviour using homotopy perturbation algorithm. The aim of this paper is to formulate and employ two computational algorithms based on homotopy perturbation method and new iterative method to obtain temperature distribution \( u(x,t) \) in a thin rod over a given finite interval whose lateral surface is not insulated.

**COMPUTATIONAL TECHNIQUES**

**Homotopy Perturbation Method (HPM)**

He (1999) proposed a new perturbation technique coupled with the homotopy technique, which is called the homotopy perturbation method (HPM). Hemeda (2012) and Falade et al. (2020) respectively applied homotopy perturbation method for solving systems of nonlinear coupled equations and solve some systems of partial differential equations, viz: the systems of coupled Burgers’ equations in one and two dimensions and the system of Laplace’s equation. Furthermore, Xiao et al. (2015) proposed a local fractional homotopy perturbation method which was an extended form of the classical homotopy perturbation method. Singh and Chatterjee (2017) presented numerical solutions of one dimensional space and time fractional advection- dispersion equation by homotopy perturbation method.

In this paper, we present the homotopy perturbation method (HPM) as a numerical technique for solving functional equation of the form:

\[
A(u) - f(\sigma) = 0, \quad \sigma \in \Omega
\]  
(6)

with boundary conditions:

\[
B \left( u, \frac{\partial u}{\partial \sigma} \right) = 0, \quad \sigma \in \Gamma
\]  
(7)
where $A$ is a general operator, $B$ is a boundary operator, $f(\sigma)$ is a known analytic function, and $\Gamma$ is the boundary of the domain $\Omega$. $A$ can further be divided into two parts $L$ and $N$ where $L$ is the linear part of $A$ and $N$ if the nonlinear part of $A$. Eq. (6) can now be expressed as:

$$L(u) + N(u) - f(\sigma) = 0, \quad \sigma \in \Omega$$

(8)

By constructing the homotopy $v(\sigma, p) : \Omega \times [0,1] \to \mathbb{R}$, which satisfies:

$$H(v, p) = (1 - p)[L(u) - L(u_0)] + p[A(v) - f(\sigma)] = 0, \quad p[0,1], \sigma \in \Omega$$

(9)

$u_0$ is the initial approximation Eq. (6), the homotopy parameter is then used to expand;

$$v = v_0 + p v_1 + p^2 v_2 + \ldots = \sum_{j=0}^{\infty} p^j v_j$$

(10)

Take $p = 1$, then

$$u(\sigma) = \lim_{p \to 1} v = \sum_{j=0}^{\infty} p^j v_j$$

(11)

**Formulation of six steps homotopy perturbation algorithm (HPA)**

In order to formulate homotopy perturbation algorithm (HPA), we consider Eq. (1) coupled with initial and boundary conditions and Eq. (6) - Eq. (11) using MAPLE 18 commands as follows:

```
restart:
Step 1:
Digits := 30;
phi := [0.1554, 0.0554];
beta := 0.3333;
P := 20;
u(x, 0) := \left\{ \begin{array}{l}
4 \sin(x) + 2 \sin(2x) + 7 \sin(3x) \\
1 - \frac{x^2}{3} \\
2 \sin(\frac{\pi x}{2}) + \sin(2\pi x) + 4 \sin(2\pi x) \\
7 \log_{10}(x)
\end{array} \right.

u_i(x, 0) := 0;
u(1, t) + u_i(1, t) := 0;
u[0] := u(x, 0) + t \times (u(1, t) + u_i(1, t));

Step 2:
for i from 0 to 0 do
A[1] := diff(u[0], x) - phi * diff(u[0], x, x) - beta * u[0];
A[2] := int(A[1], t);
u[i+1] := value(A[2]);
end do

Step 3:
for i from 1 to P do
B[1] := -phi * diff(u[i], x, x) - beta * u[i];
B[2] := -int(B[1], t);
u[i+1] := value(B[2]);
end do

Step 4:
u[HPA] := evalf(sum(u[j], j = 0..P+1));
eval(eval(u[HPA], [x = 0.0, t = 0.0]));
eval(eval(u[HPA], [x = 0.1, t = 0.1]));
eval(eval(u[HPA], [x = 0.2, t = 0.2]));
eval(eval(u[HPA], [x = 0.3, t = 0.3]));
eval(eval(u[HPA], [x = 0.4, t = 0.4]));
eval(eval(u[HPA], [x = 0.5, t = 0.5]));
eval(eval(u[HPA], [x = 0.6, t = 0.6]));
eval(eval(u[HPA], [x = 0.7, t = 0.7]));
eval(eval(u[HPA], [x = 0.8, t = 0.8]));

```

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New Iterative Method

New iterative method (NIM) was proposed by Daftardar-Gejji and Jafari (2006). NIM is simple in its principles and easy to implement on computer using symbolic computation packages such as Maple. This method is better a numerical method as it is free from rounding off errors and does not require large computer space as evident in the works of Bhalekar and Gejji (2008) and Allawee (2018).

Consider new iterative method (NIM) as a numerical technique for solving functional equation of the form

$$u(\bar{x}) = f(\bar{x}) + N(u(\bar{x}))$$  \hspace{1cm} (13)

where $N$ a nonlinear operator from a Banach space $B \rightarrow B$ and $f(\bar{x})$ is a known function.

Then $\bar{x} = (x_1, x_2, x_3, \ldots, x_n)$. We need to obtain the solution $u(\bar{x})$ of Eq. (13) having the series of the form:

$$u(\bar{x}) = \sum_{i=0}^{\infty} u_i(\bar{x})$$  \hspace{1cm} (14)

The nonlinear operator which is on the right-hand side of Eq. (13) can be decomposed as follow:

$$N \left( \sum_{i=0}^{\infty} u_i(\bar{x}) \right) = N(u_0) + \sum_{i=1}^{\infty} \left\{ N \left( \sum_{j=0}^{i} u_j \right) - N \left( \sum_{j=0}^{i-1} u_j \right) \right\}$$  \hspace{1cm} (15)

Substituting equations Eq. (13) and Eq. (14) into the equation Eq. (15) and becomes:

$$\sum_{i=0}^{\infty} u_i(\bar{x}) = f(\bar{x}) + N(u_0) + \sum_{i=1}^{\infty} \left\{ N \left( \sum_{j=0}^{i} u_j \right) - N \left( \sum_{j=0}^{i-1} u_j \right) \right\}$$  \hspace{1cm} (16)

The recurrence relation is given by

$$\begin{cases} 
    u_0 = f, \\
    u_1 = N(u_0) \\
    u_{m+1} = N(u_0 + u_1 + \cdots + u_m) - N(u_0 + u_1 + \cdots + u_{m-1}) \\
    m = 1,2,3, \\
\end{cases}$$  \hspace{1cm} (17)

Then

$$(u_1 + u_2 + \cdots + u_{m+1}) = N(u_0 + u_1 + \cdots + u_m), \quad m = 1,2,3, \ldots$$  \hspace{1cm} (18)

and

$$\sum_{i=0}^{\infty} u_i = f + N \left( \sum_{i=0}^{\infty} u_i \right)$$  \hspace{1cm} (19)

The $q$ – term approximate solution of Eq. (13) is given by;

$$u = u_0 + u_1 + \cdots + u_{q-1}$$  \hspace{1cm} (20)

Formulation of seven steps New Iterative Algorithm (NIA)

In this section, we consider Eq. (1) coupled with initial conditions (2) and Eq. (13) to Eq. (20) and formulate five steps algorithm using MAPLE 18 commands to solve Eq. (1) as follows:

```maple
eval(\(u(HPA), [x = 0.8, t = 0.8]\));
eval(\(u(HPA), [x = 0.9, t = 0.9]\));
eval(\(u(HPA), [x = 1.0, t = 1.0]\));
```

**Step 5:**

$$T[0] := eval(u(HPA), [x = 0]);$$

$$T[2] := eval(u(HPA), [x = 2]);$$

$$T[4] := eval(u(HPA), [x = 4]);$$

$$T[6] := eval(u(HPA), [x = 6]);$$

$$T[8] := eval(u(HPA), [x = 8]);$$

$$T[10] := eval(u(HPA), [x = 10]);$$

**Step 6:**

$$m[3Dplot] := plot3d(u(HPA), t = -10\pi ... 10\pi, x = -10\pi ... 10\pi, grid = [100, 100], color=[red, blue, green, yellow, black, purple], axes=BOXED, title=\text{cases});$$

$$m[2Dplot] := plot([T[0], T[2], T[4], T[6], T[8], T[10]], t = 0 ... 10, color = [red, blue, green, yellow, black, purple], axes=BOXED, title=\text{cases});$$

**Output:** Tables 1, 2, 3 and Figures: 1, 2, 3, ..., 16

where $P$ is the computational length.
restart:

Step 1:

\[ Digits := 30; \]
\[ \phi := \{0.1554, 0.0554\}; \]
\[ \beta := 0.3333; \]
\[ P := 20; \]
\[ u(x, 0) := \begin{cases} 
4 \sin(x) + 2 \sin(2x) + 7 \sin(3x) \\
1 - \frac{x^2}{3} \\
2 \sin\left(\frac{\pi x}{2}\right) + \sin(2\pi x) + 4 \sin(2\pi x) \\
7 \log_{10}(x) 
\end{cases} \]
\[ u_t(x, 0) := 0; \]
\[ u(1, t) + u_t(1, t) := 0; \]
\[ u[0] := u(x, 0) + t \times (u(1, t) + u_t(1, t)); \]

Step 2:

\[ \text{for } n \text{ from 0 to } P \text{ do} \]
\[ u[n + 1] := \text{value}(\text{int}(\phi \times \text{diff}(u[n], x, x) + \beta \times u[n], t = 0..t)); \]
\[ \text{end do} \]

Step 3:

\[ \text{for } n \text{ from 0 to } P + 1 \text{ do} \]
\[ u[n] := u[n]; \]
\[ \text{end do} \]

Step 4:

\[ \text{Sum } U := \text{sum}(u[j], j = 0 \ldots P + 1); \]
\[ \text{Simp } U := \text{simplify}(\text{sum } U); \]
\[ u[NIA] := \text{eval}(\text{simp } U); \]
\[ \text{end do} \]

Step 5:

\[ \text{eval}(u[NIA], [x = 0.0, t = 0.0]); \]
\[ \text{eval}(u[NIA], [x = 0.1, t = 0.1]); \]
\[ \text{eval}(u[NIA], [x = 0.2, t = 0.2]); \]
\[ \text{eval}(u[NIA], [x = 0.3, t = 0.3]); \]
\[ \text{eval}(u[NIA], [x = 0.4, t = 0.4]); \]
\[ \text{eval}(u[NIA], [x = 0.5, t = 0.5]); \]
\[ \text{eval}(u[NIA], [x = 0.6, t = 0.6]); \]
\[ \text{eval}(u[NIA], [x = 0.7, t = 0.7]); \]
\[ \text{eval}(u[NIA], [x = 0.8, t = 0.8]); \]
\[ \text{eval}(u[NIA], [x = 0.9, t = 0.9]); \]
\[ \text{eval}(u[NIA], [x = 1.0, t = 1.0]); \]

Step 6:

\[ T[0] := \text{eval}(u[NIA], [x = 0]); \]
\[ T[2] := \text{eval}(u[NIA], [x = 2]); \]
\[ T[4] := \text{eval}(u[NIA], [x = 4]); \]
\[ T[6] := \text{eval}(u[NIA], [x = 6]); \]
\[ T[8] := \text{eval}(u[NIA], [x = 8]); \]
\[ T[10] := \text{eval}(u[NIA], [x = 10]); \]

Step 7:

\[ m[3D plot] := \text{plot3d}(u[NIA], t = -10\pi \ldots 10\pi, x = -10\pi \ldots 10\pi, grid = [100, 100], \text{color}); \]
\[ m[2D plot] := \text{plot}([T[0], T[2], T[4], T[6], T[8], T[10]], t = 0 \ldots 10, \text{color} = \text{red, blue, green, yellow, black, purple}, \text{axes} = \text{BOXED}, \text{title} = \text{cases}); \]
\[ m[Polarpplot] := (u[NIA], t = -10\pi \ldots 10\pi, x = -10\pi \ldots 10\pi); \]
\[ m[Densityplot] := (u[NIA], t = -10\pi \ldots 10\pi, x = -10\pi \ldots 10\pi); \]

Output: Tables 1, 2, 3 and Figures 1, 2, 3, ..., 16

where P is the computational length.
Absolute error ($E_e$)

In order to determine the error involved in the proposed algorithms, we consider the absolute error as follows:

$$E_e = |u(x, t)_{exact} - u(x, t)_{numerical}|.$$  \hfill (22)

**DISCUSSION OF RESULTS**

In order to demonstrate the two proposed algorithms, we consider the thermal diffusivity of the medium under consideration as $\phi = 0.1554w/mk$ and $\beta = 0.3333$. Applying equations (12) and (21), we obtain the numerical solutions in Table 1 and Table 2.

Table 1: The temperature distribution $u(x, t)$ in a thin rod over the finite interval $I = [x|0 < x < 1]$.

<table>
<thead>
<tr>
<th>$u(x, t)$</th>
<th>Solutions</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u(x, t)$</td>
<td></td>
<td>$\phi = 0.1554$, $\beta = 0.3333$</td>
<td>$\phi = 0.0554$, $\beta = 0.3333$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$u(x, 0) = 4 \sin(x) + 2 \sin(2x) + 7 \sin(3x)$</td>
<td>$u(x, 0) = 4 \sin(x) + 2 \sin(2x) + 7 \sin(3x)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$u_t(x, 0) = 0, u(1, t) + u_t(1, t) = 0$</td>
<td>$u(1, t) + u_t(1, t) = 0$</td>
</tr>
<tr>
<td></td>
<td>Exact</td>
<td>2.652149575768369295763157416588224528</td>
<td>2.84703106140212283931233894399942570</td>
</tr>
<tr>
<td></td>
<td>HPA</td>
<td>2.652149575768369295763157416588224529</td>
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</tr>
<tr>
<td></td>
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<td>2.84703106140212283931233894399942568</td>
</tr>
<tr>
<td></td>
<td>Exact</td>
<td>4.752703401718577536973126984101481214</td>
<td>5.46016417594235243178434563700304958</td>
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<tr>
<td></td>
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<tr>
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<td>8.7531169080937989754499888891887809</td>
</tr>
</tbody>
</table>
Table 2: The temperature distribution $u(x, t)$ in a thin rod over the finite interval $I = [0 < x < 1]$.

<table>
<thead>
<tr>
<th>$u(x,t)$</th>
<th>Solutions</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
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<tbody>
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<td>$\phi = 0.1554$, $\beta = 0.3333$ $u(x,0) = 1 - \frac{x^2}{3}$, $u_t(x,0) = 0, u(1,t) + u_t(1,t) = 0$</td>
<td>$\phi = 0.0554$, $\beta = 0.3333$ $u(x,0) = 1 - \frac{x^2}{3}$, $u_t(x,0) = 0, u(1,t) + u_t(1,t) = 0$</td>
<td></td>
</tr>
<tr>
<td>$u(x,t)$</td>
<td>$hpa$</td>
<td>1.01934243971646849482336084589162</td>
<td>1.026590657437321305382318334589768</td>
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<tr>
<td></td>
<td>$n ia$</td>
<td>1.01934243971646849482336084589162</td>
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Table 3: Absolute error

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<th>Algorithm</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
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Plots’ Representation
In this section, we present behavioural pattern of temperature distribution \( u(x, t) \) in a thin rod in the 3D, 2D, polar plot, and density plots phase.

3D plots temperature distribution \( u(x, t) \) solutions in a thin rod
Polar plot temperature distribution $u(x, t)$ in a thin rod

Figure 9. Polarplot for case 1
Figure 10. Polarplot for case 2
Figure 11. Polarplot for case 3
Figure 12. Polarplot for case 4
Density plot temperature distribution $u(x,t)$ in a thin rod

CONCLUSION
The parabolic heat equation (1) was used to determine temperature distribution $u(x, t)$ in a thin rod whose lateral surface is not insulated and rod is experiencing a heat loss proportional to the difference between the rod temperature and the surrounding temperature at zero degrees. Four numerical experiments are considered to examine the relationship between coefficient $\beta$ which accounts for the heat loss in equation (1) and $\phi$ thermal diffusivity of the thin rod over the finite interval $I = \{x | 0 < x < 1\}$. The proposed algorithms have been demonstrated to be quite flexible, robust, efficiency and accurate in which the results obtained confirm the versatility, simplicity and efficiency of proposed techniques. Thus, the algorithms are suggested as good numerical tools to solve several problems in applied mathematics and other related field of sciences.

The aim of this paper is achieved from computational solutions of temperature distribution $u(x, t)$ in a thin rod which are presented in Table 1 and Table 2 for four cases of trigonometric and algebraic functions that are considered for initial temperature conditions. The following observations are made as follows:

i. Table 1 shows the numerical solutions obtained at higher the value of $\phi = 0.1554 \text{ k/sm}^2$, thermal diffusivity gives less temperature distribution $u(x, t)$ in a thin rod under the same initial and boundary conditions. Conversely, as the value $\phi = 0.0554 \text{ k/sm}^2$ decreases, thermal diffusivity gives higher temperature distribution $u(x, t)$ in a thin rod;

ii. In the same vain, Table 3 shows the numerical solutions obtained at higher the value of $\phi = 0.1554 \text{ k/sm}^2$. In this case, thermal diffusivity gives less temperature distribution $u(x, t)$ in a thin rod under the same initial and boundary conditions. In contrast, as the value of $\phi = 0.0554 \text{ k/sm}^2$ becomes less, thermal diffusivity gives higher temperature distribution $u(x, t)$ in a thin rod;

iii. The absolute error Table 3 indicates the efficiency of the two proposed algorithms;

iv. The plot representations depict the numerical solutions of temperature distribution $u(x, t)$ in a thin rod in 3D plot solutions of Figure 1, Figure 2, Figure 3 and Figure 4;

v. 2D plot solutions in Figure 5, Figure 6, Figure 7 and Figure 8 of temperatures ranges $0^\circ \leq t \leq 10^\circ$ are presented;

vi. Temperature distribution $u(x, t)$ in a thin rod is presented in polar-plots (Figure 9, Figure 10, Figure 11 and Figure 12) and the density plot is displayed in (Figure 13, Figure 14, Figure 15 and Figure 16).

Hence, the reduction of $\phi$ thermal diffusivity enhances more temperature distribution $u(x, t)$ in a thin rod.
REFERENCES


